THE EFFECT OF FLUID MOTION ON HEAT TRANSMISSION. PART I—VERTICAL CYLINDERS

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ABSTRACT. In this investigation the effect of air stream on convective heat transfer rate has been studied under different ambient conditions and the experimental data are compared with those of previous workers in this field The experimental results show that

(1) The heat transmission does not increase quite so fast as the air velocity—the rate of heat transfer being proportional to 0.517th power of air velocity ;

(2) As the characteristic dimension of the vessel is reduced its convective heat transfer per unit area per unit temperature excess increases ;

(3) For an air stream striking the cylinder at an angle of 45 degrees the heat transfer rate is about three-quarters, and for flow parallel to the axis about one-half that for flow at right angles to the axis ; there is a small variation in heat dissipation for inclinations above 60 degrees ;

(4) Clothing affects the heat transfer rate but the efficiency of insulation deteriorates with the increase in air velocity ; the double layer of clothing increases the effectiveness of insulation, but at higher velocities the protection effect of the garments is neglible in comparison with the effect of wind penetration.

A hot body in steady motion through any real fluid or at rest in a moving current loses heat by three modes of heat transfer. Losses by conduction and radiation can be minimized by suitable arrangements and under these conditions most of the heat dissipation will be due to convection—natural as well as forced. In natural or free convection the fluid motion is caused solely by gravity forces due to difference of denisty between the hotter and cooler parts, while in forced convection the fluid motion is caused by forces independent of the temperature of the fluid, such as externally imposed differences of pressure. The magnitude of forced convection depends upon the relative velocity, the physical properties of the fluid, the excess of temperature of the body over that of the fluid and the size and form of the body.

Because of the immense variety of forms of heat exchangers there exists a vast variety of experimental results. These experimental results are mostly applicable only to cases which are similar to the special arrangements employed. Numerous formulae have been, and still are, conceived, and their field of application is more or less restricted. Only very few amongst them satisfy the theoretical requirements as to their mathematical form.

In the cases of flow of hot gases through the tubes of a boiler (Nicholson, 1909), flow of liquids and gases through pipes (Stanton, 1897 ; Jordan, 1909 ; Nusselt, 1909), heat losses from honeycomb radiators, etc. , the rate of heat transfer is proportional to the n-th power of air velocity where n varies from 0.75 to unity ; while the experiments on heat dissipation from hot wires of small diameter in an air current show that the value of n is about 0.5 (Davis, 1926). Büttner (1934), interested in heat losses from human body, arrives at $n=0.52$ in experiments on convective heat losses from spheres of various sizes in a current of air. The experimental data of Winslow and others (1939) who treat the human body as a 7 cm cylinder or 15 cm sphere fits in either of the two formulae in which the value of n is 0.5 or unity. Hilpert (1933) carried out a systematic series of experiments for thin wires as well as cylindrical tubes of different diameters in air and found that his results could be expressed as

$$\text{(Nusselt's number)} = B \text{ (Renold's number)}^n$$

The values of B decrease while those of n increase with the Renold's number. They can, however, within the limits of experimental error, be treated as constant quantities in limited ranges of Renold's number.

Cylindrical bodies chosen by various workers were either thin solid wires or hollow tubes and pipes heated electrically or by some mechanical means. Horizontal cylinders have been studied in detail in both gases and liquids, but very scattered information is available for vertical cylinders. As more experimental evidence is necessary for the satisfactory understanding of the thermodynamics of heat-interchange between the body and the surroundings under different ambient conditions it was thought worthwhile to try an experiment of heat dissipation from vertical cylindrical vessels of different sizes filled with hot water and placed in a current of air under different ambient conditions in order to study the effect of fluid motion on heat transmission, and to compare the results obtained with those of previous workers in this field.

EXPERIMENTAL ARRANGEMENT

A partial theoretical solution of this problem of determining the rate of heat transfer per unit area per unit difference of temperature for a heated vessel in a gas can easily be arrived at by using the well-known method of dimensional analysis dealing with the dimensions of mass, length, time and temperature as fundamental. Based on various assumptions as to the factors involved dimensional analysis gives for forced convection the Nusselt's equation.

$$\left(\frac{1}{A\,\Delta\theta} \; \frac{dQ}{dT} \; \frac{D}{K} \right) = B' \left(\frac{VDC}{K} \right)^n \left(\frac{C\,\xi}{K} \right)^m \quad \dots \quad (1)$$

where $\frac{dQ}{dT}$ = the rate of heat transmission,

D = the characteristic dimension of the vessel,

A = area of the vessel exposed,

$\Delta\theta$ = excess of temperature of the surface of the vessel over that of the surrounding fluid,

K = thermal conductivity of the fluid,

V = fluid velocity,

C = product of the specific heat and the density of the fluid,

ξ = ratio of the viscosity of the fluid to its density,

B' = a constant,

and m, n are unassigned numbers the values of which are to be determined from experimental data.

Expression (1) can be simplified because from both kinetic theory and experiment the numeric $\frac{C\xi}{K}$ is constant for gases.

Thus $$\left(\frac{1}{A\Delta\theta}\ \frac{dQ}{dT}\ \frac{D}{K}\right)=B\left(\frac{VD}{\xi}\right)^{n} \qquad \ldots (2)$$

where B is the convection constant. The rate of heat transfer per unit area can be measured for different values of fluid velocity, characteristic dimension of the vessel, temperature excess and also for different fluids. If we plot the logarithms of the quantities in the parentheses against each other, a straight line having a slope equal to n will result.

Let us compute the effect of changing the velocity of the fluid stream alone on the heat transfer rate per unit area, all other factors remaining unchanged. Representing the logarithm of the quantity in the parenthesis on the right of equation (2) by X and that on the left by Y and expanding we have

$$X=\log D+\log V-\log\xi,$$

and $$Y=\log\left(\frac{1}{A\Delta\theta}\ \frac{DQ}{dT}\right)-\log K+\log^{n} D$$

Taking differentials for the two cases involving a change of V only we get

$$\Delta X=\Delta\log V,$$

and $$\Delta Y=\Delta\log\left(\frac{1}{A\Delta\theta}\ \frac{dQ}{dT}\right)$$

combining these, we get $$\Delta\log\left(\frac{1}{A\Delta\theta}\ \frac{dQ}{dT}\right)=\frac{\Delta Y}{\Delta X}.\ \Delta\log V \qquad \ldots (3)$$

The effect of changing the characteristic dimension of the vessel alone on the rate of heat transfer per unit area can be computed in a similar fashion. In this case we get

$$\Delta \log\left(\frac{1}{A\Delta\theta}\ \frac{dQ}{dT}\right)=\left(\frac{\Delta Y}{\Delta X}-1\right).\ \Delta \log D \qquad \dots \quad (4)$$

The experimental arrangement used by the present author and Gogate (1952) in studying the shape constant of the vessels was tried by the author in these investigations. Repetition is, therefore, avoided. Some necessary modifications were, however, made. The short focus telescope was not used, the surface temperatures of the vessels were measured by means of thermocouples and preliminary experiments were carried out in still air in order to obtain the radiation losses which also included the heat dissipation due to natural convection currents, and all the observations were corrected for these losses.

The effect of fluid motion was studied by varying the air velocity from 80 cm/sec to 1100 cm/sec. This air stream was forced on cylindrical vessels of different sizes ranging from 2 4 cm to 21.8 cm. The observations were repeated for three different values (30°, 50° and 68°C) of the excess of surface temperature of the vessel over that of the surrounding air stream. The air velocity and the rate of heat transfer were measured in the same manner (Kapadnis and Gogate, 1952) followed in their previous investigations.

The electric fan was then slowly turned round its horizontal axis by steps in order to get the beam of air current striking the vessel at different angles, thus enabling the author to study the effect of inclination of the air current on heat transmission for different air velocities.

The insulation effect due to single and double layers of clothing made up as cylindrical covers to fit the vessel properly and covering it completely was finally studied for different air velocities by measuring the heat losses in the cases of bare vessels, vessels covered with single garments and those with two.

RESULTS AND DISCUSSION

Figure 1 gives a plot of the logarithms of the quantities in the parentheses of equation (2) namely, $\log\left(\frac{1}{A\Delta\theta}\ \frac{dQ}{dT}\ \frac{D}{\xi}\right)$ against $\log\left(\frac{VD}{\xi}\right)$, which slightly concaves upwards, but is practically a straight line in all the cases of vessesls tried for different air velocities and for different values of temperature excess, a typical set of the actual observations being recorded in Table I. The slope of this line, *i.e.* the value of the index n, in these experiments, is equal to 0.517, showing clearly that the rate of heat transfer is proportional to 0.517th power of air velocity. The experimental data lie reasonably close to the curve which is represented by

$$\left(\frac{1}{A\Delta\theta}\;\frac{dQ}{dT}\;\frac{D}{K}\right)=0.56\left(\frac{VD}{\xi}\right)^{0.517} \tag{5}$$

A few points shown by the letter *b* in figure 1, however, deviate considerably from the straight line representing the logarthimic plot of equation (5). As these points correspond to velocities of the air stream higher than about ten metres per second, it means that at about this velocity of ten metres per second the linear relationship represented by the straight line (figure 1) breaks down.

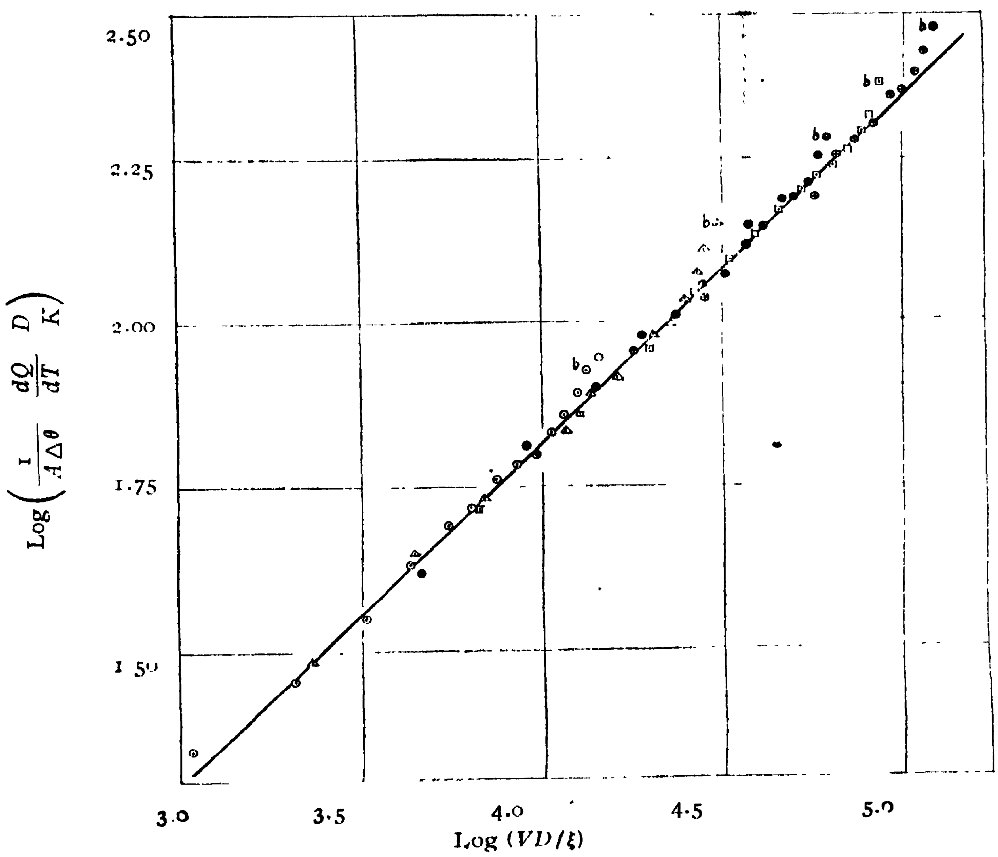

FIG. 1

Buttner (1934), in his experiments of heat losses from spheres of various sizes in a current of air, arrived at a similar result.

The experimental value of the term $\frac{\Delta Y}{\Delta X}$, *i.e.*, the slope of the straight line, found in these investigations is less than unity. Substitution in equation (3) gives

$$\Delta\log\left(\frac{1}{A\Delta\theta}\;\frac{dQ}{dT}\right)=0.517\cdot\Delta\log \quad \ldots \tag{6}$$

TABLE I

Effect of wind velocity on heat transmission in case of vessels of different sizes

Mean value of heat loss in still air ... 0.00052 cals/sec/cm²/°C

Temperature excess ... ... 30.0°C

Diameter of the vessel, cm.	ir velocity, cm/sec	Heat transfer rate cals/cm²/sec/°C,	$\frac{VD}{\xi}$	$\frac{1}{A\Delta\theta}\,\frac{dQ}{dT}\,\frac{D}{K}$
	80	5.04×10^{14}	1060	22.4
	155	6.42	2060	28.5
	242	7.95	3220	35.3
	326	9.60	4340	42.7
	405	11.08	5390	49.2
	487	11.76	6480	52.2
2.4	563	12.89	7470	57.3
	644	13.62	8570	60.5
	721	14.20	9590	63.1
	805	15.39	10700	68.4
	882	16.34	11700	72.6
	967	17.75	12900	78.9
	1020	18.98	13600	84.3
	1095	21.00	14600	93.3
	80	3.19	2310	30.7
	155	4.65	4470	44.8
	242	5.63	6970	54.2
	326	6.43	9400	61.9
	405	7.22	11700	69.5
5.2	487	8.06	14000	77.6
	563	8.38	16200	80.7
	644	9.21	18600	90.8
	721	9.94	20800	95.7
	805	10.34	23200	99.5
	882	11.13	25400	107.2
	967	12.37	27900	119.1
	1020	13.47	29400	129.7
	1095	14.67	31600	141.3
	80	2.11	4740	41.8
	155	3.35	9190	66.4
	242	4.01	14400	79.4
	326	4.82	19300	95.5
	405	5.12	24000	101.4
	487	5.66	28000	112.2
10.3	563	5.96	33400	118.0
	644	6.67	38200	132.1
	721	6.97	42800	138.0
	805	7.64	47700	151.4
	882	7.76	52300	153.8
	967	8.17	57400	161.8
	1020	8.97	60500	177.8
	1095	9.68	64900	191.9
	80	1.89	6650	52.5
	155	2.67	12900	74.1
	242	3.23	20100	89.7
	326	4.02	27100	111.7
	405	4.43	33700	123.0
	487	4.85	40500	134.6

TABLE I (contd)

Diameter of the vessel, cm	Air velocity, cm/sec	Heat transfer rate, cals/cm² sec/°C	$\frac{VD}{\xi}$	$\frac{1}{A\Delta\theta}\frac{dQ}{dT}\frac{D}{K}$
	563	5.33	46800	147.9
15.0	614	5.68	53600	157.8
	721	5.97	59900	196.0
	805	6.23	66900	173.0
	882	6.51	73300	180.7
	967	7.02	80100	105.0
	1020	7.54	84800	209.4
	1095	8.31	91000	230.7
	80	1.56	9670	63.1
	155	2.21	18700	89.1
	242	2.66	29200	107.2
	326	3.44	39400	130.0
	405	3.60	49000	145.2
	487	3.70	58800	153.1
21.8	563	4.31	68000	175.4
	644	4.01	77800	186.2
	721	4.95	87100	200.0
	805	5.44	99500	219.8
	882	5.55	107000	223.9
	967	6.02	117000	243.2
	1020	6.34	124000	255.9
	1095	6.95	132000	280.5

For a specific case, consider an air stream blowing with a speed of 326 cm/sec on the hot vessel of 15 cm diameter. For a change of V to 644 cm/sec, nearly two-fold increase, the change in the rate of heat transfer per unit area per unit temperature excess will be an increase of 1.53 fold. The calculated and the experimentally observed values for the rate of heat transfer for this specific case cited are 6.14×10^{-4} and 5.68×10^{-4} units respectively, showing a fairly good agreement within the limits of experimental error. Expressed qualitatively equation (6) means that the rate of heat transmission does not increase quite so fast as the air velocity.

Similar reasoning, applied to equation (4) which gives

$$\Delta \log \left(\frac{1}{A\Delta\theta} \frac{dQ}{dT} \right) = -0.483 \, \Delta \log D \tag{7}$$

after substitution, reveals that for a change of D from 2.4 cm to 10.7 cm, a 4.46 fold increase, there will be a 3.14 fold decrease in the rate of heat dissipation, because the term $\left(\frac{\Delta Y}{\Delta X} - 1\right)$ is negative. The experimental value of the rate of heat transmission at a velocity of 563 cm/sec for $D = 10.7$ cm is 5.66×10^{-4} units while that calculated is 4.16×10^{-4} units in this case. Equation (7) intepretted in words means that as the characteristic dimension of the vessel is reduced its convective heat transfer rate per unit area per unit excess of temperature increases.

The experimental data for these two cases is represented graphically in figure 2 in which the convective heat transfer rate is plotted as a function of air velocity for five cylinders of different sizes. With the increase in air velocity all these curves in figure 2 rise

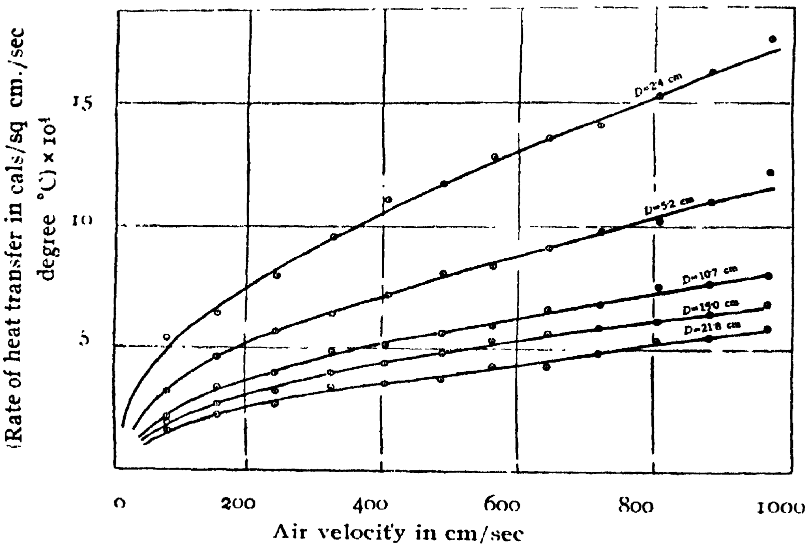

FIG. 2

steadily with a continuous decrease in slope. This is in close agreement with the interpretation of equation (6). For all the five vessels tried the heat transfer rate increases, of course, with increasing velocity, but the curves for vessels bigger in size lie below those for the smaller ones, a very good agreement, indeed, with equation (7). The importance of this fact that as the diameter of a cylinder is reduced its convective heat transfer rate per unit area per unit temperature excess increases, in the design of gas-filled incandescent lamps, is, therefore, obvious. This also suggests that, in order to measure the temperature of the fluid with a thermocuple one should prefer very fine wires. This will help increasing the ease of heat exchange between the couple and the fluid.

In figure 3, the convective heat transfer rate is plotted against the inclination of the air stream to the axis of the cylinder for different air velocities, the actual observations being given in Table II. The variation of convective heat losses for the inclinations above 60 degrees is very small. For the air stream striking the cylinder at an angle of 45 degrees the heat transfer rate is about three-quarters, and for flow parallel to the axis about one-half that for flow at right angles to the axis.

Figure 4 represents graphically the variation of the rate of heat transfer with air velocity for a bare vessel and that covered with different layers of a piece of cloth. A typical set of observations is recorded in Table III. It is found in these investigations that the wind destroys insulation as it penetrates into clothing. The efficiency of insulation deteriorates at air velocities above 500 cm/sec. It means that more clothing may have to be worn in

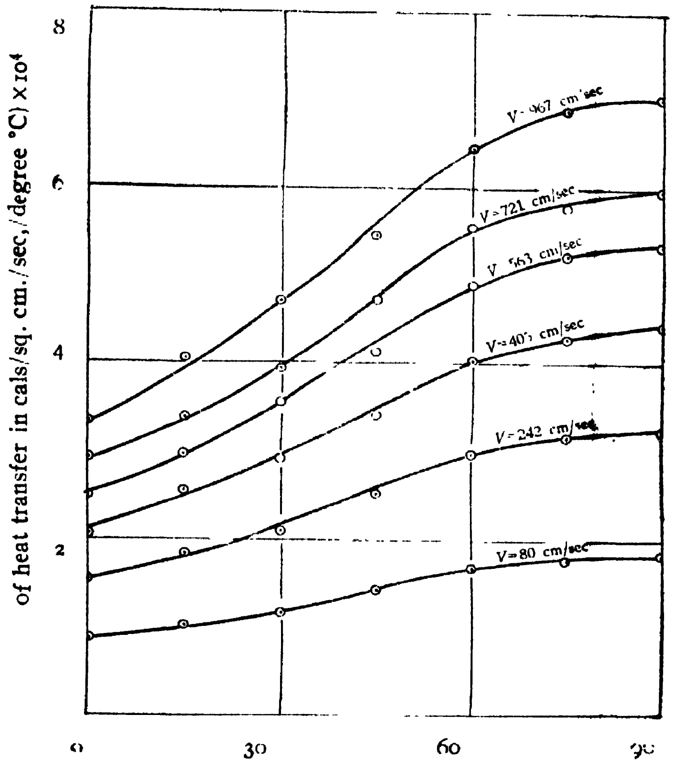

FIG. 3

TABLE II

Effect of inclination of air stream to the cylinder on heat transmission

Characteristic dimension of the vessel $= 15.0$ cm

Air velocity cm/sec	Rate of heat transfer per unit area per unit temperature excess in cals/sq. cm/sec/deg C for inclinations of						
	0°	15°	30°	45°	60°	75°	90°
	$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-4}$
80	0.93	1.08	1.25	1.49	1.74	1.85	1.89
242	1.55	1.84	2.10	2.52	2.97	3.17	3.23
405	2.08	2.57	2.92	3.42	4.03	4.30	4.43
563	2.50	2.98	3.57	4.15	4.90	5.22	5.33
721	2.93	3.41	3.94	4.72	5.56	5.80	5.97
967	3.37	4.07	4.70	5.48	6.46	6.88	7.02

the wind to furnish a specified amount of insulation. Clothing principally

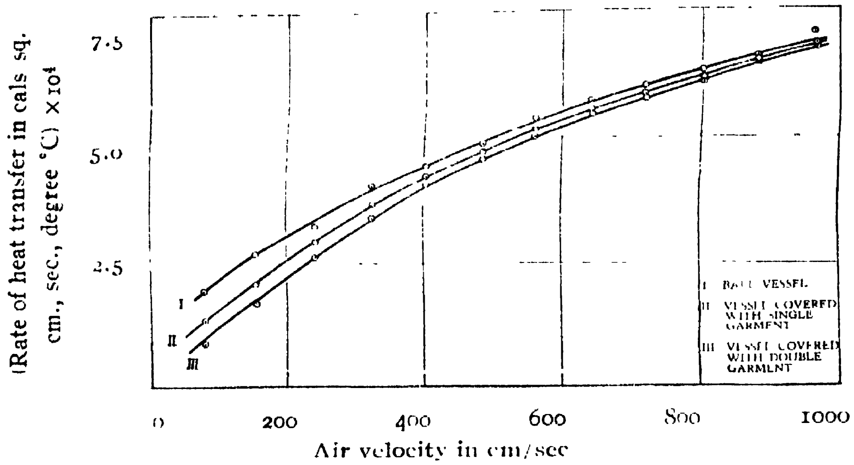

FIG. 4

affects the heat transfer. There is another physical phenomenon which is of considerable consequence in this case, and that is the tendency of the gases to form relatively thick and stable layers at surfaces, the thickness of which decreases rapidly with the increase in air velocity (Newburgh and Harris, 1945). At a velocity of about 25 metres/sec the layer is reduced to a fraction of a millimetre and is negligible as insulation.

TABLE III

Effect of insulation on heat transmission
Characteristic dimension of the vessel = 21.8 cm

Air velocity cm/sec	Rate of heat transfer per unit area per unit temperature excess in cals/sq. cm/sec/deg C for		
	bare vessel	vessel with single cover	vessel with double cover
80	1.89×10^{-4}	1.35×10^{-4}	0.85×10^{-4}
155	2.67	2.08	1.70
242	3.23	2.92	2.61
326	4.02	3.66	3.40
405	4.43	4.23	4.03
487	4.85	4.70	4.56
563	5.33	5.10	4.97
644	5.68	5.50	5.40
721	5.97	5.80	5.71
805	6.23	6.12	6.04
882	6.51	6.46	6.39
967	7.02	6.78	6.73

This layer of air, therefore, acts as insulation to the vessel—bare as well as clothed. The layers of clothing trap additional layers of air, thus, increasing the effectiveness of insulation. But at higher velocities the thickness of these insulating layers of air rapidly decreases and also the protection effect of garments becomes negligible in comparison with the effect of wind penetration. This is obvious from figure 4. The insulation effect will also depend upon the nature, size and material of the fabrics of which the piece of cloth is made.

The range of data for air flowing normal to single cylinders tried by various workers are recorded in Table IV in order to give the comparative idea of the experimental work done in this field. Figure 5 shows a logarithmic plot of the experimental data of these workers with that of the present author in the range of $\frac{VD}{\xi}$ from 10^3 to 10^5. The results of the author for Renold's numbers upto about 10^4 show somewhat higher rates of heat transmission than those of

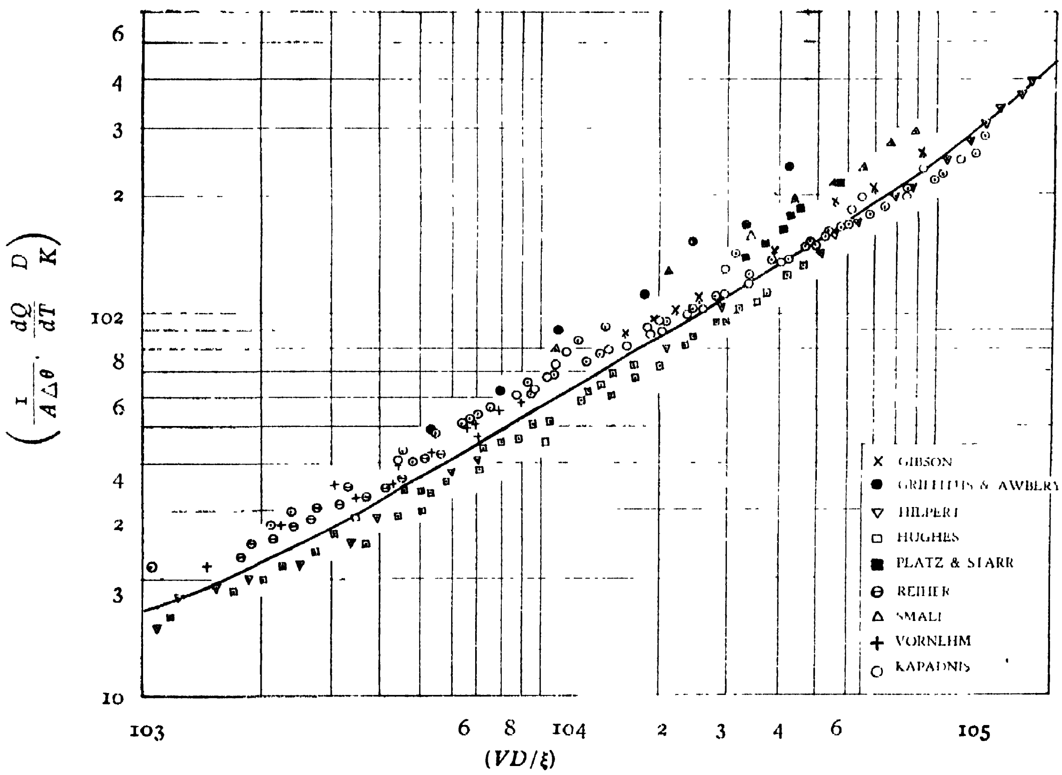

FIG. 5

other workers. The values obtained are, however in line with those of Reiher (1925) and Vornehm (1932), but hot air stream was forced on cold pipes in their experiments while the reverse was the case in the present author's experiments. Still, however, for single cylinders in the range of $\frac{VD}{\xi}$ from 10^3 to 10^5 the dimensionless equation (5) obtained from the experimenta data of the present author represents the data of various workers in this field within ± 20 %. The curve recommended by McAdams (1951) which represents the experimental data more closely is also drawn in the same figure.

TABLE IV

Range of data for air flowing normal to single cylinders
Atmospheric pressure = one atm.

Observer	Cylinder or pipe diameter in cm	Air temp. deg C	Surface temp. of pipe or cylinder deg C	Air velocity cm/sec
Gibson (1924)	9.5	10	88	271–1554
Griffiths and Awberv (1933, 1937)	3.18–8.26	18.3	28–49	76–610
Hilpert (1933)	0.0019–15.0	21	93–110	183–2957
Hughes (1916)	0.43–5 50	15.6	100	0–1524
Paltz and Starr (1931)	8.26	21	100	700–1220
Reiher (1925)	1.5–2 8	260	21–35	274–579
Small (1935)	11.4	21	77–92	152–1219
Vornehm (1932)	2.41–3 05	199	27–41	122–762
Present Author	2.4–21 8	30	60–98	8–1095

This work is expected to throw some light on the understanding of the thermodynamics of the heat-interchange between the body and the surroundings under different ambient conditions.

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